Limit theorems and wrapping transformations in bi-free probability theory

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In classical probability, Lévy and Khintchine demonstrated that the limit law associated with any triangular array of infinitesimal random variables is infinitely divisible. In this talk, we shall manifest the analogous results for distributions on the plane and bi-torus in the framework of bi-free probability theory. Like the classical situation, bi-freely additive and multiplicative infinitely divisible distributions, and solely these distributions serve as the limiting distributions of a triangular array of infinitesimal random variables. The bi-free harmonic analysis developed by ourselves performs an essential role in the study of bi-free limit theorems. These limit theorem consequences also establish tight bonds between classical and bi-free probability theories. If time permits, some other relevant topics will be discussed.

Invariant measure and flow associated to the Φ^4-quantum field model on the three-dimensional torus

Seiichiro Kusuoka
(Kyoto University)

We give a direct construction global flows for the stochastic quantization equation to the quantum field theoretical Φ^4-model on the 3-dimensional torus. For the construction we prepare stationary processes of approximation equations of the stochastic quantization equation and show the tightness of approximation processes. The limit of marginal distributions is also stationary with respect to the limit process and is regarded as a Φ^4_3-measure on the torus. In this sense, we introduce a new construction of a Φ^4_3-measure. For the proof we use the method of singular stochastic partial differential equations, Besov spaces and paraproducts.

Asymptotic behavior of random walk in cooling random environment

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One-dimensional Random Walk in Cooling Random Environment (RWCRE) is obtained as a patchwork of one-dimensional Random Walk in Random Environment (RWRE) by resampling the environment along a sequence of deterministic times. The RWCRE model can be seen as a model that interpolates between the classical static model and the model with i.i.d. resamplings every unit of time.

In this talk, we have two results about the asymptotic behavior of RWCRE. First, we investigate how the recurrence versus transience criterion known for RWRE changes for RWCRE. Second, we explore the fluctuation for RWCRE when RWRE is either recurrent or satisfies a classical central limit theorem. In the previous work, we showed that SLLN and LDP for RWCRE were the same as those for RWRE under a certain condition for the resampling. However, two results in this talk are different from those for RWRE. They really depend in a delicate way on how we choose resampling. In particular, sub-diffusive scaling and convergence to mixtures of different limit laws are possible.

This talk is based on a joint work with L. Avena, C. da Costa and F. den Hollander (Leiden University).

Keywords: random walk in random environment, random walk in dynamical random environment, asymptotic behavior

References

[1] L. Avena, F. den Hollander, Random walks in cooling random environments, arXiv e-prints (2017), arXiv:1610.00641.

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Gaussian fluctuations in two-dimensional surface growth models

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In this talk, I will discuss stochastic surface growth dynamics within the anisotropic class of the Kardar–Parisi–Zhang (KPZ) equation. Fluctuations of the dynamics are conjectured to be Gaussian in the limit of large time as if an unusual nonlinear term in the equation does not exist. The first proofs, proven recently, study iterated scaling limits of some particle systems. I will discuss these results and explain the particular roles of the de Moivre–Laplace theorem and the Poisson limit theorem for the problem.

Monte Carlo Markov Processes and Related Problems

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Abstract: Monte Carlo Markov processes have been widely used to calculate the expectations of statistics or to approximate the underlying probability when direct sampling is not feasible. The evaluation of the approximation depends on various criteria, e.g. asymptotic variance, spectral gap, convergence exponent in variational norm etc. This is a survey of our work. Open problems will be discussed.

Random walk conditioned on survival among Bernoulli obstacles: sub-critical phase

Ryoki Fukushima
(Research Institute for Mathematical Sciences, Kyoto University)

I will present two recent results on a discrete time random walk conditioned to avoid Bernoulli obstacles on the d-dimensional integer lattice obtained in joint works with Jian Ding, Rongfeng Sun and Changji Xu. The first result on this model dates back to a famous work by Donsker and Varadhan on the Wiener sausage in 1975. Since then, it has been intensively studied and various localization results have been proved. In particular, the random walk is known to localize in a ball of sub-diffusive size under the annealed law. Our first result gives a more detailed geometric description of the range of the random walk. More precisely, we showed that it completely fills the ball where the walk is localized, and in addition we got a sharp estimate on the size of its boundary.

Our second result is about the response to an external force. If we give a bias to the random walk, then the model is known to undergo a phase transition: for a large bias, the walk is ballistic whereas for a small bias, it is sub-ballistic. This phase transition was proved by Sznitman and later, Ioffe and Velenik studied the ballistic phase in detail. In the sub-ballistic phase, physicists conjectured that the walk is localized in a sub-diffusive scale as in the unbiased case, but it has not been proved. We prove this conjecture with a precise information on the behavior of whole path.

Optimal Markov Chain Monte Carlo Sampling

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In this talk, I will introduce optimal Markov chain Monte Carlo (MCMC) sampling. The focus is on homogeneous Markov chains. I will first define the problem of finding the optimal transition matrix based on minimizing the asymptotic variance. Results on three types of optimal transition matrices [1, 2, 3] will be presented, and one open problem will be mentioned. I will also discuss the locally optimal sampler (LOS): an MCMC sampling that performs local updates based on the optimal transition matrix. In our simulation studies, the LOS was shown to provide an improved rate of convergence over the MetropolisVHastings and the Gibbs Sampler. The implementation of the LOS requires only minor modifications in existing Gibbs sampling code.

Keywords: Markov chain Monte Carlo, asymptotic variance, transition matrix

References

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[3] Lu-Jing Huang, Yin-Ting Liao, Ting-Li Chen, and Chii-Ruey Hwang, Optimal variance reduction for Markov chain Monte Carlo, *SIAM Journal on Control and Optimization*, 56.4 (2018): 2977-2996.

Automatic Sleep Scoring by the Scattering Transform, Diffusion Maps, and Hidden Markov models

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This presentation discusses an unsupervised approach for sleep dynamics exploration and automatic annotation by combining modern harmonic analysis tools. We apply a nonlinear-type time frequency analysis tool to extract the frequency-domain features from a pair of physiological signals. The dynamics of the spectral information is visualized by the multiview diffusion maps. Based on the feature sequence and the expert-determined sleep stages, we construct a hidden Markov model to predict the sleep stages of new subjects. The prediction performance is validated on a publicly available benchmark database, Physionet Sleep-EDF SC and ST, with the leave-one-subject-out cross validation. This talk includes the joint works [1, 2] with Yu-Lun Lo, John Malik, Yuan-Chung Sheu, and Hau-Tieng Wu.

Keywords: multiview diffusion maps, hidden Markov models.

References

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[2] G. R. Liu, Y. L. Lo, Y. C. Sheu, H. T. Wu, Explore intrinsic geometry of sleep dynamics and predict sleep stage by unsupervised learning techniques, to be appeared in the Springer book *Harmonic Analysis and Applications* https://arxiv.org/pdf/1905.04589.pdf